B.Sc. Part-I

Paper-I

Theory of Relativity

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B. Sc. Part 1 Physics Date 30.4.21

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Dept of Physics HDJC

Theory of Relativity

Problem :- A Particle of mass m_1 moving with velocity u makes a head on collision with a Particle of mass m_2 initially at rest. So that their final velocities v_1 & v_2 are along the same line. If the collision is elastic, show that $v_2 = \frac{2u}{1 + m_2/m_1}$ and also calculate the fraction of total energy shared by m_2.

Soln – According to Principle of conservation of momentum.

Momentum before input = Momentum after input

$$m_1 u + m_2 \cdot 0 = m_1 v_1 + m_2 v_2$$

or $$m_1 u = m_1 v_1 + m_2 v_2 \quad \text{——} (1)$$

According to Principle of Conservation of energy

energy before impact = energy after impact

$$\frac{1}{2} m_1 u^2 + \frac{1}{2} m_2 \cdot 0 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

or $$\frac{1}{2} m_1 u^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad \text{——} (2)$$

Since in elastic collision, there is no loss of kinetic energy.

∴ from eqn (1) $m_1 v_1 = m_1 u - m_2 v_2$ —— (3)

or $$v_1 = \frac{m_1 u - m_2 v_2}{m_1} \quad \text{——} (4)$$

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Substituting this value of v_1 in equ^{n} (2) we get

$$\frac{1}{2} m_1 u^2 = \frac{1}{2} m_1 \left(\frac{m_1 u - m_2 v_2}{m_1} \right)^2 + \frac{1}{2} m_2 v_2^2$$

$$\text{or} \quad m_1 u^2 = \frac{m_1 (m_1 u - m_2 v_2)^2}{m_1^2} + m_2 v_2^2$$

$$\text{or} \quad m_1 u^2 = \frac{(m_1 u - m_2 v_2)^2 + m_1 m_2 v_2^2}{m_1}$$

$$\text{or} \quad m_1^2 u^2 = (m_1 u - m_2 v_2)^2 + m_1 m_2 v_2^2$$

$$\text{or} \quad m_1^2 u^2 = m_1^2 u^2 + m_2^2 v_2^2 - 2 m_1 m_2 u v_2 + m_1 m_2 v_2^2$$

$$\text{or} \quad m_1^2 u^2 - m_1^2 u^2 = m_2^2 v_2^2 - 2 m_1 m_2 u v_2 + m_1 m_2 v_2^2$$

$$\text{or} \quad 0 = m_2^2 v_2^2 - 2 m_1 m_2 u v_2 + m_1 m_2 v_1^2$$

Now dividing by $m_2 v_2$ we get

$$0 = \frac{m_2^2 v_2^2 - 2 m_1 m_2 u v_2 + m_1 m_2 v_2^2}{m_2 v_2}$$

$$0 = \frac{m_2^2 v_2^2}{m_2 v_2} - \frac{2 m_1 m_2 u v_2}{m_2 v_2} + \frac{m_1 m_2 v_2^2}{m_2 v_2}$$

$$0 = m_2 v_2 - 2 m_1 u + m_1 v_2$$

$$\text{or} \quad 2 m_1 u = m_2 v_2 + m_1 v_2$$

$$2 m_1 u = (m_1 + m_2) v_2$$

$$\therefore \quad v_2 = \frac{2 m_1 u}{m_1 + m_2}$$

$$= \frac{2 m_1 u}{m_1} \Big/ \frac{m_1 + m_2}{m_1}$$

$$= \frac{2u}{1 + \frac{m_2}{m_1}}$$

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or $$\boxed{v_2 = \frac{2u}{1 + \frac{m_2}{m_1}}} \quad \text{——— (5)}$$

This Proves the 1st Part

for 2nd Part —

Fraction of energy shared by m_2

$$= \frac{\frac{1}{2} m_2 v_2^2}{\frac{1}{2} m_1 u^2} = \frac{m_2 v_2^2}{m_1 u^2}$$

$$= \frac{m_2 \left\{ \frac{2u}{1 + m_2/m_1} \right\}^2}{m_1 u^2}$$

$$= \frac{m_2 \left\{ \frac{2u}{\frac{m_1 + m_2}{m_1}} \right\}^2}{m_1 u^2}$$

$$= \frac{m_2 \left\{ \frac{2u m_1}{m_1 + m_2} \right\}^2}{m_1 u^2}$$

$$= \frac{4 m_2 m_1^2 u^2}{(m_1 + m_2)^2 / m_1 u^2} = \frac{4 m_2 m_1^2 u^2}{(m_1 + m_2)^2} \cdot \frac{1}{m_1 u^2}$$

$$= \frac{4 m_1 m_2}{(m_1 + m_2)^2}$$

Proved

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